Design of State Feedback Controller for a Quadruple Tank Process

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Abstract- Most of the large and complex industrial processes are naturally Multi Input Multi Output systems. MIMO systems in comparison with SISO systems are difficult to control due to inherent nonlinearity and due to the existence of interactions among input and output variables. Control of nonlinear MIMO process is cumbersome because nonlinear process does not obey superposition and homogeneity property. This paper presents an implementation of decentralized PID controller and pole placement controller to quadruple tank process with two input and two output model. The process is firstly decoupled through a stable simplified de coupler to attain the benefits of decentralized control techniques. Then, a single input single output PID controller tuning method is used to determine optimal PID controllers for each loop. Finally, performance of the designed controller is measured by the simulation.

Index terms- Multivariable Process; Decentralised PID; state feedback controller.

1. INTRODUCTION

In general for SISO systems different techniques of controller design are proposed and practiced in reality. Now a day's most of the complex systems are MIMO system. Decentralized (or multi-loop) PID control systems (Chen and Seborg, 2003) are widely used for MIMO control problems in spite of the development of advanced control strategies such as MPC and IMC. In practice, design and implementation of controllers for MIMO system is a difficult task due to the coupling and interactions between inputs and outputs. In last few decades, designing controllers for MIMO systems has attracted lot of researchers. The control objective is to control each output independently, in spite of changes in manipulated or load variables. Lee et al. proposed the analytical design method of decentralised PID controller for MIMO systems [3]. The design of optimally tuned decentralised PID controller using real coded genetic algorithm is proposed by Donghai Li.et.al.[2]

The proposed work is presented as follows. The section II gives state space modelling of quadruple tank process. In Section III the concepts of design of decoupler is discussed. Section IV explains about the concepts of design of decentralised PID controller and pole placement controller. The simulated results are shown in section V. The work is concluded in section VI.

2. PROCESS DESCRIPTION

A nonlinear mathematical model for the four-tank system is obtained by simple mass balance according to Bernoulli's equation (Gatzke et al., 2000). The schematic diagram of quadruple tank Process is shown in Fig.1. The differential equations representing the mass balances in this four-tank system are given by equation [1]

$$\frac{dh_1}{dt} = -\frac{a_1}{A_1}\sqrt{2gh_1} + \frac{a_3}{A_1}\sqrt{2gh_3} + \frac{\gamma_1 k_1}{A_1}v_1$$
$$\frac{dh_2}{dt} = -\frac{a_2}{A_2}\sqrt{2gh_2} + \frac{a_4}{A_2}\sqrt{2gh_4} + \frac{\gamma_2 k_2}{A_2}v_2$$
$$\frac{dh_3}{dt} = -\frac{a_3}{A_3}\sqrt{2gh_3} + + \frac{(1-\gamma_2)k_2}{A_3}v_2$$
$$\frac{dh_4}{dt} = -\frac{a_4}{A_4}\sqrt{2gh_4} + \frac{(1-\gamma_1)k_1}{A_4}v_1 \quad [1]$$

Where h_i *is* the liquid level in tank *i*; a_i is the cross sectional area of connecting pipe of tank *i*; A_i is the cross sectional area of tank *i*; v_j is the voltage setting of pump *j*, with the corresponding gain k_j ; γ_j is the portion of the flow into the upper tank from pump *j*; The process-manipulated inputs are v_1 and v_2 (voltage settings of the pumps) and the measured outputs are γ_1 and γ_2 (voltages from level measurement and signal conditioning devices). The measured level signals are assumed to be proportional to the true levels, *i.e.*, $\gamma_1 = k_{m1}h_1$ and $\gamma_2 = k_{m2}h_2$. The level sensors are calibrated so that $k_{m1} = k_{m2} = 1$. This simple mass balance model adopts Bernoulli's law for flow out of the orifice. Johansson and Nunes (1998) have shown that the inverse response in the modelled outputs will occur when $\gamma_1 + \gamma_2 < 1$. The nominal operating condition for a minimum phase system is given in Table. *I*

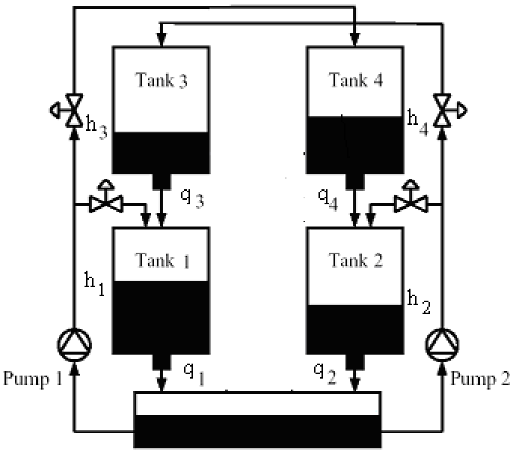

Fig.1. Schematic diagram of quadruple tank Process

Table1. Nominal Operating conditions and Parameter values

Symbol	State/Parameters	Minimum phase system
$h_{1s}, h_{2s}, h_{3s}, h_{4s}$	Nominal levels	[12.4; 12.7; 1.8; 1.4] *cm*
v_{1s}, v_{2s}	Nominal Pump settings	[3.00;3.00] *cm*
ai	Area of the drain in Tank *i*	[0.071;0.057; 0.071;0.057] cm^2
Ai	Areas of the tanks	[28;32; 28;32] cm^2
γ_1	Ratio of flow in Tank 1 to flow in Tank 4	0.7
γ_2	Ratio of flow in Tank 2 to flow in Tank 3	0.6
kj	Pump proportionality constants	[3.33; 3.35] $cm^3/V\ sec$
g	Gravitational constant	981 cm/sec^2

The non-linear model equations are linearized using Taylor series approximation and the state space model thus obtained is given by equation [2].

$$\begin{bmatrix} \dot h_1 \\ \dot h_2 \\ \dot h_3 \\ \dot h_4 \end{bmatrix} = \begin{bmatrix} -\frac{1}{\tau_1} & 0 & \frac{A_3}{A_1\tau_3} & 0 \\ 0 & -\frac{1}{\tau_2} & 0 & \frac{A_4}{A_2\tau_4} \\ 0 & 0 & -\frac{1}{\tau_3} & 0 \\ 0 & 0 & 0 & -\frac{1}{\tau_4} \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \end{bmatrix} + \begin{bmatrix} \frac{\gamma_1 k_1}{A_1} & 0 \\ 0 & \frac{\gamma_2 k_2}{A_2} \\ 0 & \frac{(1-\gamma_2)k_2}{A_3} \\ \frac{(1-\gamma_1)k_1}{A_4} & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \end{bmatrix} \qquad [2]$$

The state space model at the nominal operating parameter values for minimum phase system is obtained and given by equation [3]

$$\begin{bmatrix} \dot h_1 \\ \dot h_2 \\ \dot h_3 \\ \dot h_4 \end{bmatrix} = \begin{bmatrix} -0.016 & 0 & 0.042 & 0 \\ 0 & -0.011 & 0 & 0.033 \\ 0 & 0 & -0.042 & 0 \\ 0 & 0 & 0 & -0.033 \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \end{bmatrix} + \begin{bmatrix} 0.08 & 0 \\ 0 & 0.063 \\ 0 & 0.048 \\ 0.03 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \qquad [3]$$

The corresponding linear transfer function matrix is:

$$G(s) = \begin{bmatrix} \frac{\gamma_1 c_1}{\tau_1 s + 1} & \frac{(1-\gamma_2)c_2}{(\tau_1 s + 1)(\tau_3 s + 1)} \\ \frac{(1-\gamma_1)c_1}{(\tau_2 s + 1)(\tau_4 s + 1)} & \frac{\gamma_2 c_2}{\tau_2 s + 1} \end{bmatrix}$$

For this minimum phase system, the time constants are:

$$\tau_i = \frac{A_i}{a_i}\sqrt{\frac{2h_{is}}{g}} \quad \text{and} \quad c_j = \frac{\tau_j k_j}{A_j}, \ j = 1,2$$

Substituting the nominal operating parameters values given in table 1, the transfer function matrix is obtained and given by equation [4].

$$\begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix} = \begin{bmatrix} \frac{2.6}{(62s+1)} & \frac{1.5}{(62s+1)(23s+1)} \\ \frac{1.4}{(30s+1)(90s+1)} & \frac{2.8}{(90s+1)} \end{bmatrix} \quad [4]$$

3. DE COUPLER DESIGN

The Relative Gain Array (RGA) is first calculated to measure the amount of interaction and to decide the type of pairing (Luyben and Luyben, 1997). Then decentralized PID controller is designed for the system using ZN tuning method.
The steady state gain matrix of the system is,

$$G(0) = \begin{bmatrix} g_{11}(0) & g_{12}(0) \\ g_{21}(0) & g_{22}(0) \end{bmatrix} = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix}$$

Multiplying the individual elements of $G(0)$ and $G^{-1}(0)$ yields a RGA matrix,

$$\Lambda = \begin{bmatrix} \frac{k_{11}k_{22}}{k_{11}k_{22}-k_{12}k_{21}} & \frac{-k_{12}k_{21}}{k_{11}k_{22}-k_{12}k_{21}} \\ \frac{-k_{21}k_{12}}{k_{11}k_{22}-k_{12}k_{21}} & \frac{k_{11}k_{22}}{k_{11}k_{22}-k_{12}k_{21}} \end{bmatrix} = \begin{bmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{21} & \lambda_{22} \end{bmatrix}$$

The RGA matrix of the minimum phase system is,

$$RGA = \begin{bmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{21} & \lambda_{22} \end{bmatrix} = \begin{bmatrix} 1.81 & -0.81 \\ -0.81 & 1.81 \end{bmatrix}$$

The RGA indicates that the loop pairing y_1 with u_1 and y_2 with u_2 is appropriate.
In simplified decoupling, the decoupling matrix is represented to the form

$$D(s) = \begin{bmatrix} 1 & d_{12}(s) \\ d_{21}(s) & 1 \end{bmatrix}$$

Here, a decoupled response and the de coupler are specified with the structure in the equation given below.

$$G_p(S)D(S) = \begin{bmatrix} g_{11}^{eff}(s) & 0 \\ 0 & g_{22}^{eff}(s) \end{bmatrix}$$

$$\begin{bmatrix} g_{11}(s) & g_{12}(s) \\ g_{21}(s) & g_{22}(s) \end{bmatrix} \begin{bmatrix} 1 & d_{12}(s) \\ d_{21}(s) & 1 \end{bmatrix} = \begin{bmatrix} g_{11}^{eff}(s) & 0 \\ 0 & g_{22}^{eff}(s) \end{bmatrix}$$

Solving the above matrix we get,

$$d_{12}(s) = -\frac{g_{12}(s)}{g_{11}(s)} = \frac{-0.025}{S+0.04}$$

$$d_{21}(s) = -\frac{g_{21}(s)}{g_{22}(s)} = \frac{-0.017}{s+0.033}$$

The decoupling elements must be stable and should be physically realizable.

4. CONTROLLER DESIGN

4.1. ZN PID controller

A PID controller is widely used in all industrial process. A PID controller takes action based on the value of error. The three mode controller adjusts the manipulated variable so as to minimize error. A decoupling decentralized controller is used. The conventional ZN PID controller tuning parameters are obtained using auto tuner PID block in Matlab simulink control design tool. The designed PID parameters are given in Table.2. The schematic diagram of a MIMO plant with simplified de coupler and PID controller is shown in Fig.2.

Table 2: ZN PID parameters:

Controller Type	Quadruple tank process		
		Loop 1	Loop 2
ZN PID controller	kc	0.84	0.72
	k_i	0.04	0.02
	k_d	-4.5	-8.1

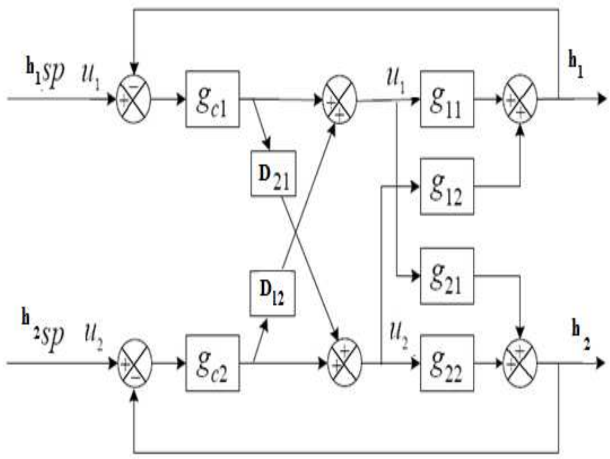

Fig.2. Schematic diagram of a TITO process with simplified decoupler and PID controller.

4.2. Pole placement controller

The concept of feed-backing all the state variables back to the input of the system through a suitable feedback gain matrix in the control strategy is known as the full-state variable feedback control technique. Using this approach, the desired location of the closed-loop Eigen values (poles) of the system will be specified. Thus, the aim is to design a feedback controller that will move some or all of the open-loop poles of the measured system to the desired closed-loop pole location as specified. Hence, this approach is often known as the pole-placement control design. In order to perform the pole-placement design technique, the system must be a "completely state controllable". Normally, the major disadvantage in the design of the state feedback controller using pole-placement is large steady-state error. Therefore, in order to compensate for this problem, an integral control is added which eliminates the steady-state error in the response to the step input. The simulink diagram of state feedback control for MIMO system is shown in Fig.3.

The control law is, $u = K\left[r(t) - x(t)\right]$

Now, the state feedback gain matrix is obtained by solving the equation given below

$$\det\left(\left[(sI - A) + (B)K\right]\right)$$
$$= (s - p_1)(s - p_2)(s - p_3)(s - p_4) = 0$$

Where p_1,p_2 p_3 and p_4 are arbitrary pole locations chosen to obtain reasonable speed and damping in the transient response.

The poles are chosen as

$$p = [-0.07 \pm j0.07, -0.3, -0.02]$$

The state feedback controller gains are,

$$k = \begin{bmatrix} 3.17 & -0.45 & -0.56 & -0.54 \\ -1.94 & 4.8 & -3.85 & 2.45 \end{bmatrix}$$

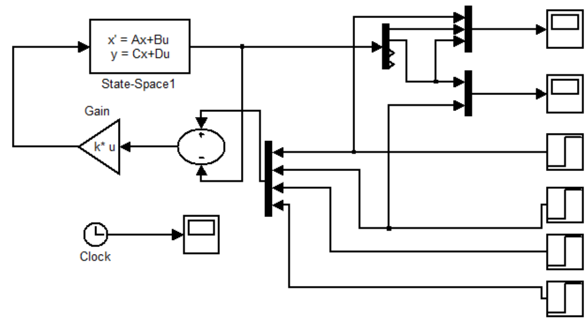

Fig.3. Simulink diagram of a MIMO plant with state feedback controller

5. SIMULATION RESULTS

In order to analyze the performance of the proposed controllers, the system is simulated using MATLAB/SIMULINK. The open loop response of the designed system is shown in Fig.4 and which indicates that, the open loop system is stable but set point tracking is not obtainable. The servo response of ZN tuned PID controller in comparison with state feedback controller is shown in Fig.5 & Fig.6 respectively for step change in pump voltage. In order to demonstrate the disturbance rejection capability of control schemes, simulation studies have been carried by applying a disturbance input at *t=200* sec and ends at *t=400* seconds. The servo regulatory response of the proposed controller is shown in Fig.7& Fig.8 respectively. The regulatory response shows that, the state feedback controller is less sensitive to disturbances.

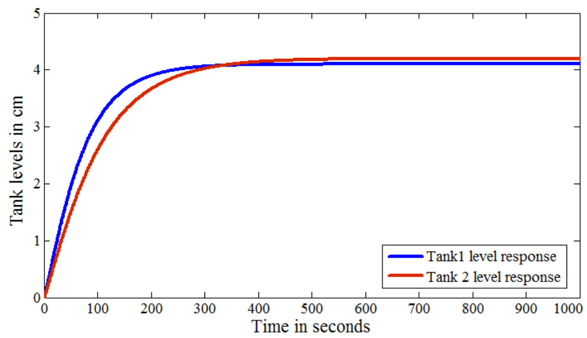

Fig.4. Open loop response of tank Levels for step input

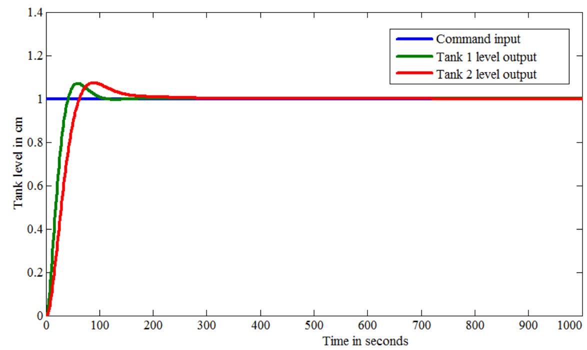

Fig.5. Servo response for decentralized PID Controller

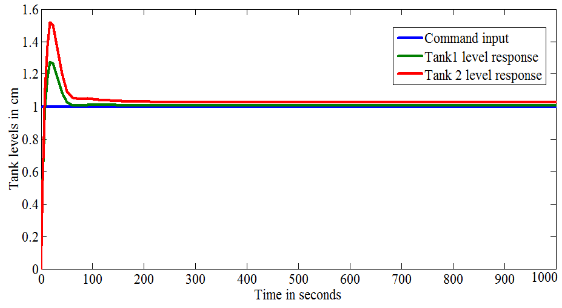

Fig.6. Servo response of state feedback controller

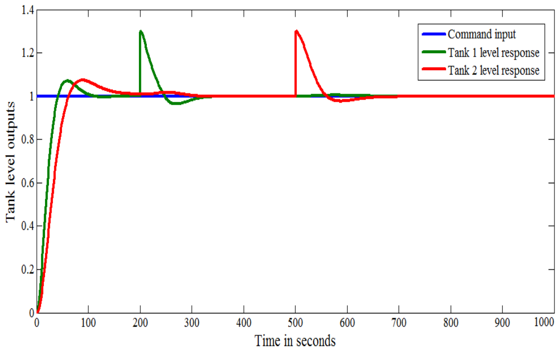

Fig.7. Servo regulatory response for decentralized PID controller

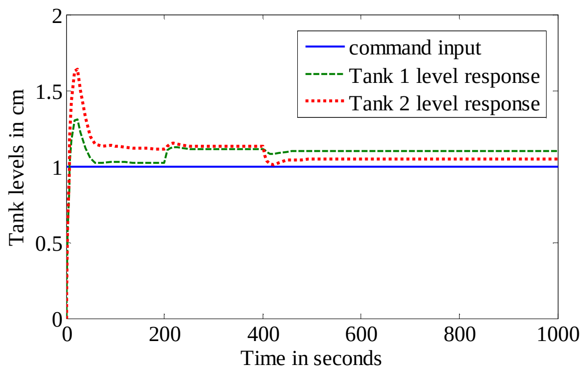

Fig.8. Servo regulatory response of state feedback Controller

6. CONCLUSION

The proposed algorithm is demonstrated for a nonlinear Quadruple tank process. It has been shown that, state feedback controller offers better control performance than PI controller. The performance summary is given in Table.3 and which indicates that, the performance of the State feedback controller far exceeds the PID controller's performance with respect to settling time, peak time and steady state error. The future scope of the work is that, optimization techniques may be used for finding the state feedback gains which ensures superior performance.

Table 3: Comparison of Time domain specifications & Performance indices of quadruple tank process

Parameter	Conventional PID controller		State feedback controller	
	Loop1	Loop2	Loop1	Loop2
t_r in sec	26	40	**5.6**	**2.5**
t_s in sec	172	408	**50.4**	**134**
$\%M_p$	7	7.33	17	17
tp in sec	59	87	**26.5**	**48**
ISE (servo)	12.8	20.6	**8.37**	**3.4**
IAE (servo)	51.5	34.4	**44.4**	**16.8**

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